06 - Normal distribution

HCI/PSYCH 522
Iowa State University

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Overview

- Normal distribution
  - Continuous (non-count) data

Normal

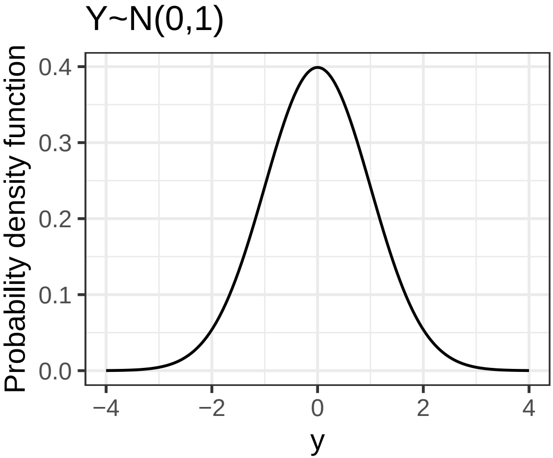
We typically model numerical data with a normal distribution. If $Y \sim N(\mu, \sigma^2)$, then

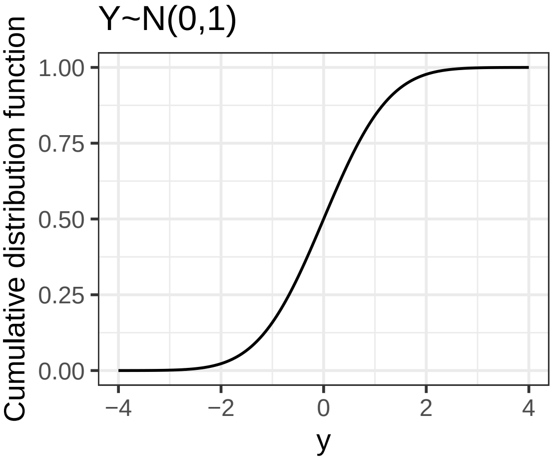
- the expected value $E[Y] = \mu$,
- variance $Var[Y] = \sigma^2$,
- standard deviation $SD[Y] = \sigma$,
- probability density function (bell-shaped curve)

$$f(y) = (2\pi\sigma^2)^{-1/2} \exp\left(-\frac{1}{2\sigma^2}(y-\mu)^2\right),$$

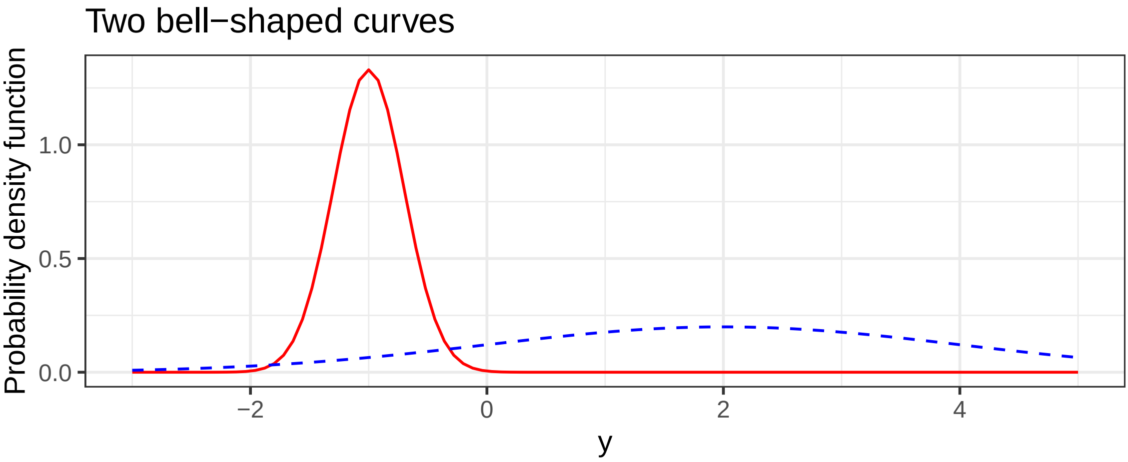
- cumulative distribution function $P(Y \leq y)$.

Normal

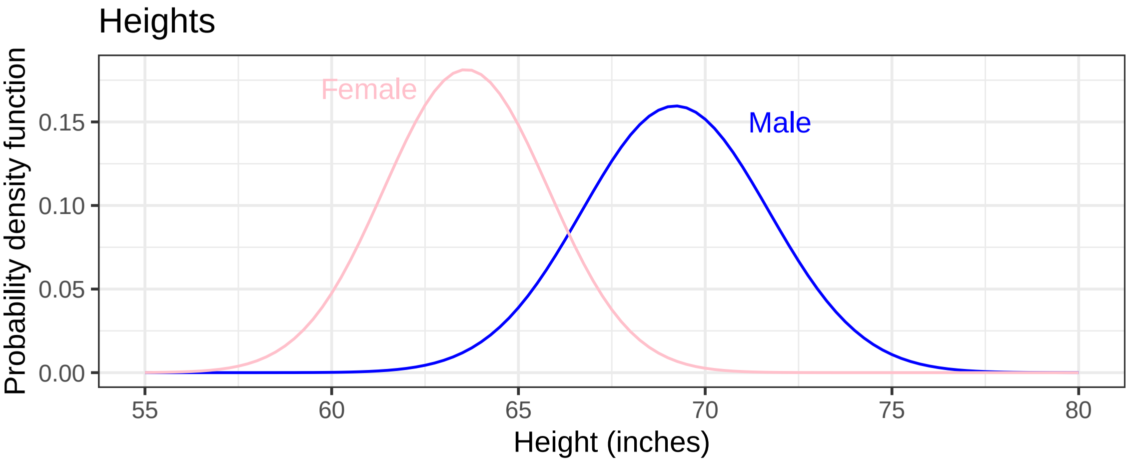

Y~N(0,1)
Probability density function
0.4
0.3
0.2
0.1
0.0
−4
−2
0
2
4
y


Y~N(0,1)
Cumulative distribution function
1.00
0.75
0.50
0.25
0.00
−4
−2
0
2
4
y

Normal

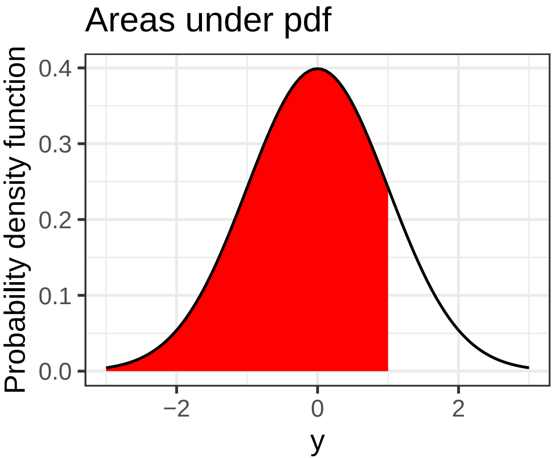

Two bell−shaped curves
Probability density function
1.0
0.5
0.0
−2
0
2
4
y

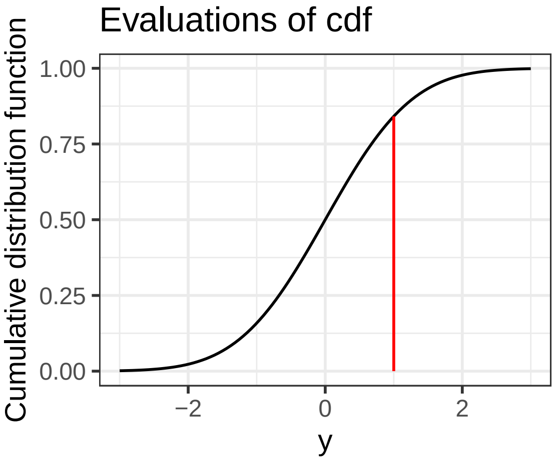
Heights


Heights
Female
Male
Probability density function
0.15
0.10
0.05
0.00
55
60
65
70
75
80
Height (inches)

Probabilities

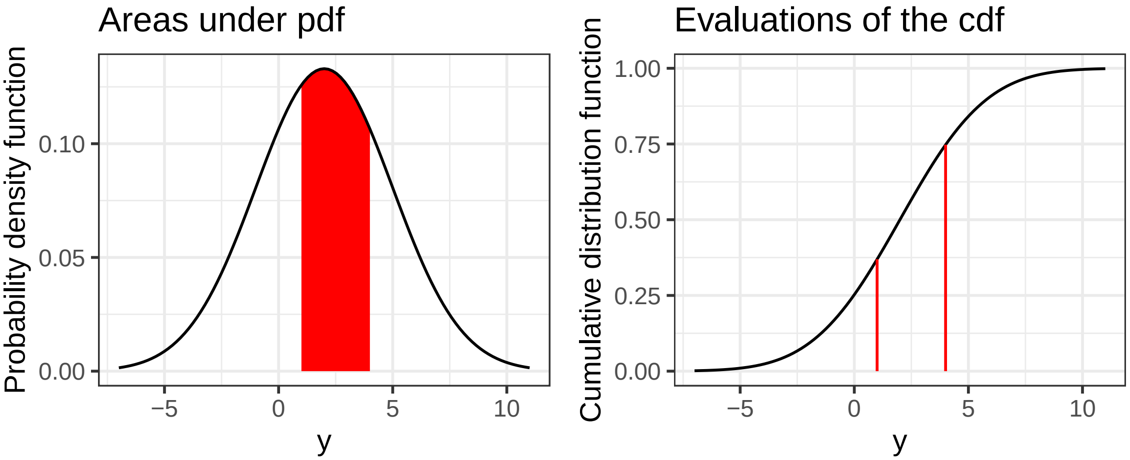
Let $Y \sim N(0,1)$ and calculate $P(Y < 1)$.





Probabilities

Let $Y \sim N(2, 3^2)$ and calculate $P(1 < Y < 4) = P(Y < 4) - P(Y < 1)$.



Probabilities in R

Let $Y \sim N(-3, 4^2)$.

```
mn <- -3
s  <- 4
```

Calculate $P(Y < 0)$.

```
pnorm(0, mean = -3, sd = 4)

## [1] 0.7733726
```

Calculate $P(Y > 1)$.

```
1-pnorm(1, mean = -3, sd = 4)

## [1] 0.1586553
```

Probabilities in R

Let $Y \sim N(-3, 4^2)$.

```
mn <- -3
s  <- 4
```

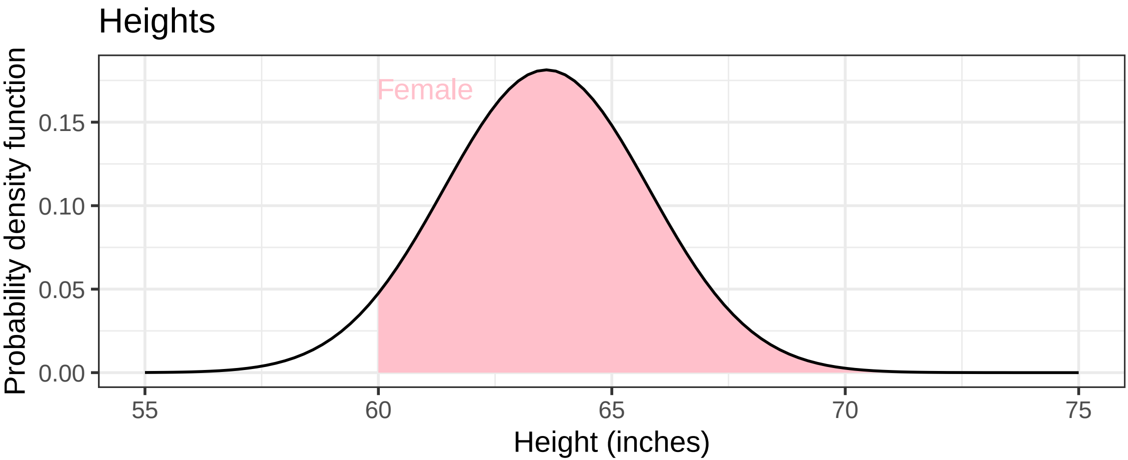
Calculate $P(0 < Y < 1) = P(Y < 1) - P(Y < 0)$.

```
pnorm(1, mean = -3, sd = 4) - pnorm(0, mean = -3, sd = 4)

## [1] 0.0679721
```

For continuous random variables, e.g. normal, $P(Y = y) = 0$ for any value y. This is NOT true for discrete random variables, e.g. binomial.

Probability female height is above 60 inches?



Probability female height is above 60 inches?

Let $Y \sim N(63.6, 2.2^2)$. Calculate $P(Y > 60)$.

```
1-pnorm(60, mean = 63.6, sd = 2.2)

## [1] 0.9491182
```