1. Let Z have a standard normal distribution. Calculate the following probabilities.

Answer:

We write $Z \sim N(0,1)$. See page 3 for areas under the curve depicting these probabilities.

(a) $P(Z < 0.67)$

Answer:

$$P(Z < 0.67) \quad = 0.7486 \quad z\text{-table}$$

(b) $P(Z > 1.32)$

Answer:

$$\begin{array}{lll} P(Z > 1.32) & = 1 - P(Z \leq 1.32) & \text{probabilities sum to 1} \\ & = 1 - P(Z < 1.32) & \text{continuous} \\ & = 1 - 0.9066 & z\text{-table} \\ & = 0.0934 & \text{calculation} \end{array}$$

(c) $P(-1.10 < Z < 0.49)$

Answer:

$$\begin{array}{lll} P(-1.10 < Z < 0.49) & = P(Z < 0.49) - P(Z < -1.10) & \\ & = 0.6879 - 0.1357 & z\text{-table} \\ & = 0.5522 & \text{calculation} \end{array}$$

2. Let Y be a normal random variable with mean -3 and standard deviation 4. Calculate the following probabilities.

Answer:
We write $Y \sim N(-3, 4^2)$.

(a) $P(Y < -5)$

Answer:

$$\begin{aligned} P(Y < -5) &= P\left(\tfrac{Y-(-3)}{4} < \tfrac{-5-(-3)}{4}\right) && \text{standardize} \\ &= P(Z < -0.50) && \text{calculation} \\ &= 0.3085 && z\text{-table} \end{aligned}$$

(b) $P(Y > -9)$

Answer:

$$\begin{aligned} P(Y > -10) &= 1 - P(Y \leq -10) && \text{probabilities sum to 1} \\ &= 1 - P(Y < -10) && \text{continuous} \\ &= 1 - P\left(\tfrac{Y-(-3)}{4} < \tfrac{-10-(-3)}{4}\right) && \text{standardize} \\ &= 1 - P(Z < -1.75) && \text{calculation} \\ &= 1 - 0.0401 && z\text{-table} \\ &= 0.9599 && \text{calculation} \end{aligned}$$

(c) $P(0 < Y < 1)$

Answer:

$$\begin{aligned} P(0 < Y < 1) &= P(Y < 1) - P(Y \leq 0) && \\ &= P(Y < 1) - P(Y < 0) && \text{continuous} \\ &= P\left(\tfrac{Y-(-3)}{4} < \tfrac{1-(-3)}{4}\right) - P\left(\tfrac{Y-(-3)}{4} < \tfrac{0-(-3)}{4}\right) && \text{standardize} \\ &= P(Z < 1) - P(Z < 0.75) && \text{calculation} \\ &= 0.8413 - 0.7734 && z\text{-table} \\ &= 0.0679 && \text{calculation} \end{aligned}$$

Answer:

All the probabilities correspond to areas under the associated bell curve. For problem 1), this is the standard normal curve, i.e. centered at 0 with standard deviation of 1. For problem 2), this is the normal curve associated with a normal with mean -3 and standard deviation of 4. The areas associated with all the problems are shown below.