1. A company manufacturing bolts produces bolts that weigh 0.86 grams (g) on average with a variance of 0.04 g^2. To verify that bolts are being produced as expected, 72 bolts are weighed together and their sample mean is computed.

   Answer:
   Let X_i be the weight (g) for bolt i. Assume the bolts are independent and identically distributed with mean $E[X_i] = \mu = 0.86$ g and standard deviation $\sigma = 0.2$ g. We have a sample size of $n = 72$ and thus the CLT applies.

   (a) What is the expected value of the sample mean?
   Answer:

   $$E[\overline{X}_n] = 0.86 \text{ g.}$$

   (b) What is the standard error of the sample mean?
   Answer:

   $$SE[\overline{X}_n] = 0.2/\sqrt{72} = 0.02357 \text{ g.}$$

   (c) What is the approximate probability the sample mean is greater than 0.87 g?
   Answer:

   $$\begin{aligned} P(\overline{X}_n > 0.87) &= P\left(\overline{X}_n > 0.87\right) \\ &= P\left(\tfrac{\overline{X}_n - 0.86}{0.02357} > \tfrac{0.87-0.86}{0.02357}\right) \\ &\approx P(Z > 0.42) \\ &= 1 - P(Z < 0.42) \\ &= 1 - 0.6628 = 0.3372 \end{aligned}$$

2. Daily sales for a grocery store follow an unknown distribution with mean \$10T (T=thousand) and standard deviation \$5T. (Hint: the following questions ask about the total sales where the total sales are just n times the average daily sales.)

Answer:
Let X_i be the sales for day i for the month of April with $1, \dots, n = 30$. We are given that $E[X_i] = \$10T$ and $Var[X_i] = \$5^2T^2$. The CLT tells us that $\overline{X} \dot{\sim} N(\$10T, \$5^2T^2/30)$. The following questions all involve the total sales which we will denote as S and note that $S = \sum_{i=1}^{n} X_i = n\overline{X}$.

(a) What is the expected total sales for the month of April?

Answer:

$$E[S] = E\left[n \cdot \overline{X}\right] = n \cdot E\left[\overline{X}\right] = 30 \times \$10T = \$300T.$$

(b) What is the standard error for the total sales for the month of April?

Answer:
As the standard error is the square root of the variance, we will calculate the variance first.

$$Var[S] = Var\left[n \cdot \overline{X}\right] = n^2 \cdot Var\left[\overline{X}\right]$$

We can calculate this as we know $n = 30$ and $Var\left[\overline{X}\right] = \sigma^2/n$. The standard error is then

$$\sqrt{Var[S]} = \sqrt{n^2 \cdot Var\left[\overline{X}\right]} = n\sqrt{Var\left[\overline{X}\right]} = n \cdot SE[\overline{X}] = 30 \cdot 5/\sqrt{30} = \sqrt{30} \cdot 5 = 27.38613.$$

(c) In order to break even in April, the store needs at least \$280T in total sales. What is the approximate probability the store will NOT break even?

Answer:
This problem can be solved by realizing that $S \dot{\sim} N(n\mu, n\sigma^2)$ where $n\mu$ is the answer to part a) and $\sqrt{n} \cdot \sigma$ is the answer to part b). Here we will solve the problem by turning it into a problem about the sample mean.

$$P(S < 280) = P(S/n < 280/30) = P(\overline{X} < 9.33) = P\left(\frac{\overline{X} - 10}{5/\sqrt{30}} < \frac{9.33 - 10}{5/\sqrt{30}}\right) \approx P(Z < -0.73) = 0.2643$$

where the last result arises from looking the probability up on a z-table. Thus, there is a 26% probability the store will not break even.