1. The Threaded Screw Products Co., Inc. makes #8 wood screws that are intended to have a mean torque strength of 150 in-lbs. Engineers at the company routinely test screws in a destructive process to ensure the mean torque strength is at least 150 in-lbs. In the most recent batch, the engineers tested 50 random screws and found a sample mean torque strength of 149.5 in-lbs. They know the standard deviation around the mean is 8.3 in-lbs.

Answer:
Let X_i be the torque strength of screw i with $E[X_i] = \mu$ is unknown and $SD[X_i] = 8.3$ in-lbs. The sample size is $n = 50$ with $\overline{x} = 149.5$ in-lbs. By the CLT, $\overline{X} \dot{\sim} N(\mu, \sigma^2/n)$.

   (a) Construct confidence intervals for the population mean torque strength for the following confidence levels using the Empirical Rule.

   Answer:
   All of the following confidence intervals require $\sigma/\sqrt{n} = 8.3/\sqrt{50} = 1.173797$.

      i. 68%

      Answer:
      By the Empirical Rule, the CI is

$$\overline{x} \pm \frac{\sigma}{\sqrt{n}} = 149.5 \pm 1.173797 = (148.3 \text{ in-lbs}, 150.7 \text{ in-lbs}).$$

      ii. 95%

      Answer:
      By the Empirical Rule, the CI is

$$\overline{x} \pm 2 \cdot \frac{\sigma}{\sqrt{n}} = 149.5 \pm 2 \cdot 1.173797 = (147.2 \text{ in-lbs}, 151.8 \text{ in-lbs}).$$

      iii. 99.7%

      Answer:
      By the Empirical Rule, the CI is

$$\overline{x} \pm 3 \cdot \frac{\sigma}{\sqrt{n}} = 149.5 \pm 1.173797 = (146.0 \text{ in-lbs}, 153.0 \text{ in-lbs}).$$

   (b) Construct confidence intervals for the population mean torque strength for the following confidence levels.

      i. 75%

      Answer:
      Since $C = 75\%$, $\alpha = 0.25$, $\alpha/2 = 0.125$, and $z_{\alpha/2} = 1.15$.

$$\overline{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 149.5 \pm 1.15 \cdot 1.173797 = (148.2 \text{ in-lbs}, 150.8 \text{ in-lbs}).$$

      ii. 90%

      Answer:
      Since $C = 90\%$, $\alpha = 0.10$, $\alpha/2 = 0.05$, and $z_{\alpha/2} = 1.645$.

$$\overline{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 149.5 \pm 1.645 \cdot 1.173797 = (147.6 \text{ in-lbs}, 151.4 \text{ in-lbs}).$$

      iii. 99%

      Answer:
      Since $C = 99\%$, $\alpha = 0.01$, $\alpha/2 = 0.005$, and $z_{\alpha/2} = 2.575$.

$$\overline{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 149.5 \pm 2.575 \cdot 1.173797 = (148.2 \text{ in-lbs}, 150.8 \text{ in-lbs}).$$

2. Iowa State University would like to understand student debt when graduating from the University. The take a random sample of 80 graduating seniors and find that their average debt is \$17,500. The University knows the standard deviation around the average debt is \$8,000. Construct an 85% confidence interval for the mean student debt.

Answer:
Let X_i be the debt for student i and $E[X_i] = \mu$ is unknown while $SD[X_i] = \$8,000$. The sample size is $n = 80$ with $\overline{x} = \$17,500$. For an 85% confidence interval, we have $\alpha = 0.15$, $\alpha/2 = 0.075$, and $z_{\alpha/2} = 1.44$. Thus the 85% confidence interval for the mean debt is

$$\overline{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = \$17,500 \pm 1.44 \cdot \frac{\$8,000}{\sqrt{80}} = (\$16,212, \$18,788).$$

3. Proctor & Gamble is trying to understand usage of its Old Spice deodorant. They recruited a random sample of 121 customers to record how many days they use Old Spice in the coming year. At the conclusion of the survey, the average number of days a customer used Old Spice in the year was 205 days. P&G knows that the variance around the population mean is 196 days^2. Construct a 97% confidence interval for the mean number of days Old Spice is used across its customers.

Answer:
Let X_i be the number of days individual i used Old Spice in the year with $E[X_i] = \mu$ being unknown, $Var[X_i] = 196 \text{ days}^2$, and $SD[X_i] = \sqrt{196} = 14$ days. For a 97% confidence interval, we have $\alpha = 0.03$, $\alpha/2 = 0.015$, and $z_{\alpha/2} = 2.17$. Thus a 97% confidence interval for the mean number of days customers used Old Spice is

$$\overline{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 205 \pm 2.17 \cdot \frac{14}{\sqrt{121}} = (202 \text{ days}, 208 \text{ days}).$$