1. Find the following t critical values, $t_{n-1,\alpha/2}$, for the following values of n and confidence level C:

   Answer:
   The t-table contains confidence level at the bottom, nonetheless, I will convert to $\alpha/2$ for notational purposes. If the exact df and/or confidence level does not exist in the t-table, then take the largest critical value.

   (a) $n = 6$ and $C = 99\%$

   Answer:
   $t_{5,0.005} = 4.032$

   (b) $n = 31$ and $C = 70\%$

   Answer:
   $t_{30,0.15} = 1.055$

   (c) $n = 40$ and $C = 99.9\%$

   Answer:
   The degrees of freedom here are 39, but only 30 and 40 degrees of freedom are on the table. The lower degrees of freedom will always have the larger critical value, so we use 30 degrees of freedom to calculate the critical value. $t_{39,0.0005} < t_{30,0.0005} = 3.646$

   (d) $n = 1000$ and $C = 85\%$

   Answer:
   The degrees of freedom here are 999, but only 100 and 1000 degrees of freedom are on the table. Similarly the confidence level is 85% but only 80% and 90% are on the table. The lower degrees of freedom will always have the larger critical value, so we use 100 degrees of freedom to calculate the critical value. Similarly the higher confidence level will always have the larger critica value, so we will sue 90% confidence. $t_{999,0.075} < t_{100,0.1} = 1.660$

2. The Threaded Screw Products Co., Inc. makes #8 wood screws that are intended to have a mean torque strength of 150 in-lbs. Engineers at the company routinely test screws in a destructive process to ensure the mean torque strength is at least 150 in-lbs. In the most recent batch, the engineers tested 50 random screws and found a sample mean torque strength of 149.5 in-lbs and sample standard deviation of 8.3 in-lbs.

   Answer:
   Let X_i be the torque strength of screw i with $E[X_i] = \mu$ and $SD[X_i] = \sigma$ both unknown. The sample size is $n = 50$ with $\overline{x} = 149.5$ in-lbs and $s = 8.3$ in-lbs.

   Assuming the torque strengths are normally distributed, construct confidence intervals for the population mean torque strength for the following confidence levels.

   Answer:
   The degrees of freedom are 49 which doesn't exist in the table. Thus we will use 40 degrees of freedom to find all critical values. For all of the following we will

   (a) 70%

   Answer:
   The critical value is $t_{40,0.15} = 1.05$ The confidence interval is

$$\overline{x} \pm t_{40,0.15} \cdot \frac{s}{\sqrt{n}} = 149.5 \pm 1.05 \cdot \frac{8.3}{\sqrt{50}} = (148.3 \text{ in-lbs}, 150.7 \text{ in-lbs}).$$

(b) 95%

Answer:

The critical value is $t_{40,0.025} = 2.021$ The confidence interval is

$$\overline{x} \pm t_{40,0.025} \cdot \frac{s}{\sqrt{n}} = 149.5 \pm 2.021 \cdot \frac{8.3}{\sqrt{50}} = (147.1 \text{ in-lbs}, 151.9 \text{ in-lbs}).$$

(c) 81%

Answer:

As 81% confidence level is not in the table, we will use 90% confidence. The critical value is $t_{40,0.05} = 1.684$ The confidence interval is

$$\overline{x} \pm t_{40,0.05} \cdot \frac{s}{\sqrt{n}} = 149.5 \pm 1.684 \cdot \frac{8.3}{\sqrt{50}} = (147.5 \text{ in-lbs}, 151.5 \text{ in-lbs}).$$

3. Iowa State University would like to understand student debt when graduating from the University. ISU takes a random sample of 80 graduating seniors and find that their average debt is \$17,500 and the standard deviation is \$8,000. Assuming debt is normally distributed, construct an 80% confidence interval for the mean student debt.

   Answer:
   Let X_i be the debt for student i with $E[X_i] = \mu$ and $SD[X_i] = \sigma$ both unknown. The sample size is $n = 80$ with $\overline{x} = \$17,500$ and $s = \$8,000$. For an 80% confidence interval, we have $t_{79,0.075} < t_{60,0.1} = 1.296$. Thus an 80% confidence interval for the mean debt is

$$\overline{x} \pm t_{60,0.1} \frac{s}{\sqrt{n}} = \$17,500 \pm 1.296 \cdot \frac{\$8,000}{\sqrt{80}} = (\$16,341, \$18,659).$$

4. Proctor & Gamble is trying to understand usage of its Old Spice deodorant. They recruited a random sample of 121 customers to record how many days they use Old Spice in the coming year. At the conclusion of the survey, the average number of days a customer used Old Spice in the year was 205 days and the variance was 196 days^2. Assuming days used is normally distributed, construct a 96% confidence interval for the mean number of days Old Spice is used last year.

   Answer:
   Let X_i be the number of days individual i used Old Spice in the year with $E[X_i] = \mu$ and $SD[X_i] = \sigma$ both unknown. With a sample size of $n = 121$, we observed a sample mean of $\overline{x} = 205$ days and a sample standard deviation of $s = \sqrt{194} = 14$ days. For a 96% confidence interval, we have $t_{120,0.02} < t_{100,0.02} = 2.081$. Thus a 96% confidence interval for the mean number of days customers used Old Spice is

$$\overline{x} \pm t_{100,0.02} \frac{s}{\sqrt{n}} = 205 \pm 2.081 \cdot \frac{14}{\sqrt{121}} = (202 \text{ days}, 208 \text{ days}).$$