Midterm review

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Iowa State University

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What we have covered

Chapters

- Probability and inference (Ch 1)

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- Single-parameter models (Ch 2)

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- Decision theory (Sec 9.1)

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  - Inference via simulations

Single-parameter models (Ch 2)

General

- Priors
  - Conjugate (Sec 2.4)
  - Default - Jeffreys (Sec 2.8)
  - Weakly informative (Sec 2.9)
- Posteriors
  - Compromise between data and prior (2.2)
  - Point estimation
  - Credible intervals (Sec 2.3)

Specific models

- Binomial (Sec 2.1–2.4)
- Normal, unknown mean (Sec 2.5)
- Normal, unknown variance (Sec 2.6)
- Poisson (Sec 2.6)
- Exponential (Sec 2.6)
- Poisson with exposure (Sec 2.7)

Single-parameter models (Ch 2)

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  - Highest posterior density

Introduction to multiparameter models (Ch 3)

- Joint posterior

$$p(\theta_1, \ldots, \theta_n | y) \propto p(y|\theta_1, \ldots, \theta_n) p(\theta_1, \ldots, \theta_n)$$

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- Conditional posteriors

$$p(\theta_2, \ldots, \theta_n|\theta_1, y) \propto p(\theta_1, \ldots, \theta_n|y)$$

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- Posterior decomposition, e.g.

$$p(\theta_1, \ldots, \theta_n|y) = p(\theta_1|y) \prod_{i=2}^{n} p(\theta_i|\theta_{1:i-1}, y)$$

where $1 : i-1 = 1, 2, \ldots, i-1$.

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- Conditional independence, e.g.

$$p(\theta_i | \theta_{1:i-1}, y) = p(\theta_i | \theta_{i-1}, y)$$

Normal model

- Normal model with default prior (Sec 3.2)

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- Point estimation:
  $$\hat{\theta}_{Bayes} \to \hat{\theta}_{MLE} \stackrel{p}{\to} \theta_0$$
- Limiting distribution:
  $$\theta|y \stackrel{d}{\to} N\left(\hat{\theta}, \frac{1}{n}\mathrm{I}(\hat{\theta})^{-1}\right)$$

Asymptotics - What can go wrong?

- Not unique to Bayesian statistics
  - Unidentified parameters
  - Number of parameters increase with sample size
  - Aliasing
  - Unbounded likelihoods
  - Tails of the distribution
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    - Convergence to the edge of the parameter space

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- Hierarchical normal model (Sec 5.4)

$$y_{ij} \stackrel{iid}{\sim} N(\mu_j, \sigma_j^2) \quad \mu_j \stackrel{iid}{\sim} N(\eta, \tau^2) \quad \sigma_j^2 \stackrel{iid}{\sim} Ga(\alpha, \beta)$$

Model checking (Ch 6)

- Data replications

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- Graphical posterior predictive checks (Sec 6.4)
- Posterior predictive pvalues (Sec 6.3)

$$p_B = P(T(y^{rep}, \theta) \geq T(y, \theta)|y)$$

for a test statistic $T(y, \theta)$.

Hypothesis testing (Section 7.4)

From a Bayesian perspective,

$$\text{Simple: } H_i : \theta = \theta_i \qquad \text{Composite: } H_i : \theta \in (\theta_i, \theta_{i+1}]$$

Treat all simple (or all composite) hypotheses as formal Bayesian parameter estimation. Treat a mix of simple and composite hypotheses as formal Bayesian tests.

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Formal Bayesian tests

- require prior probabilities for each hypothesis, $p(H_i)$,

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Treat all simple (or all composite) hypotheses as formal Bayesian parameter estimation. Treat a mix of simple and composite hypotheses as formal Bayesian tests.

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- the marginal likelihood, $p(y|H_i)$.

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where $p(\theta)$ represents your current state of belief, i.e. it could be a prior or a posterior depending on your perspective.

Stan

```
model = "
data {
  int<lower=0> N;
  int<lower=0> n[N];
  int<lower=0> y[N];
  real s;
}
parameters {
  real<lower=0,upper=1> mu;
  real<lower=0> eta;
}
transformed parameters {
  real<lower=0> alpha;
  real<lower=0> beta;
  alpha <- eta *    mu;
  beta  <- eta * (1-mu);
}
model {
  mu  ~ beta(20,30);
  eta ~ lognormal(0,s);
  y   ~ beta_binomial(n,alpha,beta);
}
generated quantities {
  real<lower=0,upper=1> theta[N];
  for (i in 1:N) theta[i] <- beta_rng(alpha+y[i], beta+n[i]-y[i]);
}
"
```