Name _____

Fall 2024 **STAT 5870-1/A** **Exam I**
(100 points)

Instructions:

- Write your name on the top, but do not open the exam.
- You are allowed to use one 8.5" x 11" page of notes (front and back) and a calculator.
- A total of 4 pages with a front and back.
- For full/partial credit, show all your work.

1. Identify the most appropriate distribution to model the data described. For each question, **circle one** of binomial, Poisson, or normal. (2 pts each)

   (a) The height of a tree in Sycamore row

   binomial Poisson normal

   (b) Number of musical notes played incorrectly in a measure with 9 notes

   binomial Poisson normal

   (c) Number of coding bugs in the R package 'emmeans'

   binomial Poisson normal

   (d) Time taken to complete a 5k

   binomial Poisson normal

   (e) Number of songs Taylor Swift will sing off her Evermore album at her next concert

   binomial Poisson normal

   (f) Amount of money donated to Story Theatre Company this year

   binomial Poisson normal

   (g) Score for an Olympic dive

   binomial Poisson normal

   (h) In a survey of 50 people, the number who use liquid detergent when washing clothes

   binomial Poisson normal

   (i) Calories consumed in a day

   binomial Poisson normal

   (j) Number of times you would clap when Dr. Niemi wins the Iowa State University Teaching Excellence Award

   binomial Poisson normal

2. Consider the following probability density function (pdf)

$$f(x) = \begin{cases} (x-1)/2 & 1 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Explain why this is a valid pdf. (4 pts)

(b) State the image for a random variable with this probability density function. (4 pts)

(c) Determine the cumulative distribution function for a random variable with this pdf. (8 pts)

(d) State the integral that would be used to find the mean for a random variable with this pdf. More points will be rewarded for an integral this is more specific, but you do NOT need to solve the integral for full credit. (4 pts)

3. In 2022, a mother tested positive for opiates shortly after giving birth to her baby. Data from 2013 suggests that only 6 of every 1,000 pregnant mothers use opiates. For the test used, the sensitivity (the probability of testing positive if opiates have been used) is 95% while the specificity (the probability of testing negative if opiates have not been used) is 85%. Based on this positive test, what is the probability this mother did use opiates. (20 pts)

4. Carbon has two stable, non-radioactive isotopes, ^{12}C and ^{13}C, with relative isotopic abundances of 98.89% and 1.11%, respectively. The molecular formula for cholesterol is $C_{27}H_{44}O$, i.e. there are 27 carbon atoms.

   (a) What is the mean number of ^{13}C atoms in a molecule of cholesterol? (4 pts)

   (b) What is the standard deviation for the number of ^{13}C atoms in a molecule of cholesterol? (4 pts)

   (c) What is the probability that a molecule of cholesterol has no ^{13}C atoms? (4 pts)

   (d) What is the probability that a molecule of cholesterol has at least one atom of ^{13}C? (4 pts)

   (e) If we want to assure the probability of at least one atom of ^{13}C in a molecule of cholesterol is less than 0.05, what would the relative isotropic abundance of ^{13}C need to be? (4 pts)

5. The Ames air quality monitor is set to collect $PM_{2.5}$ (particulate matter less than 2.5 μm) once per day and report the total amount of pollutant measured after 30 days. Currently, the average $PM_{2.5}$ measurement in Ames is 9.9 $\mu g/m^3$ per day with a standard deviation of 4.9 $\mu g/m^3$ per day.

   (a) What is the expected sum of Ames $PM_{2.5}$ measurements over the next 30 days? (4 pts)

   (b) Assuming measurements are independent, what is the variance of the sum of Ames $PM_{2.5}$ measurements over the next 30 days? (4 pts)

   (c) What is the approximate probability the 30-day **average** is greater than the national standard of $9\,\mu g/m^3$? (8 pts)

   (d) What would the expected daily Ames $PM_{2.5}$ measurement need to be so that the probability of the 30-day average being greater than $9\,\mu g/m^3$ is 16%? (4 pts)

Please use as scratch paper.

Table 1: Cumulative distribution function, $P(Z \leq z)$, for standard normal

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998