I05a - Sampling distribution

STAT 5870 (Engineering)
Iowa State University

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Sampling distribution

The sampling distribution of a statistic is the distribution of the statistic *over different realizations of the data*.

Find the following sampling distributions:

- If $Y_i \stackrel{ind}{\sim} N(\mu, \sigma^2)$,

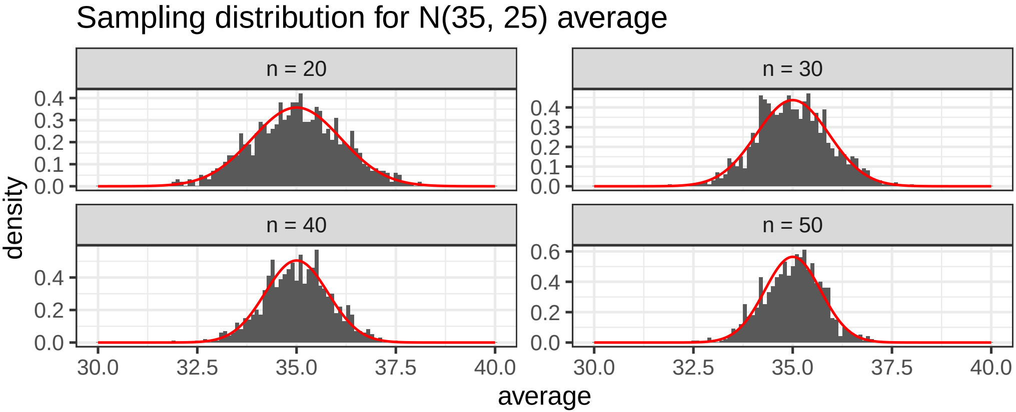
$$\overline{Y} \qquad \text{and} \qquad \frac{\overline{Y} - \mu}{S/\sqrt{n}}.$$

- If $Y \sim Bin(n, p)$,

$$\frac{Y}{n}.$$

Normal model

Let $Y_i \stackrel{ind}{\sim} N(\mu, \sigma^2)$, then $\overline{Y} \sim N(\mu, \sigma^2/n)$.



Normal model

Let $Y_i \stackrel{ind}{\sim} N(\mu, \sigma^2)$, then the t-statistic

$$T = \frac{\overline{Y} - \mu}{S/\sqrt{n}} \sim t_{n-1}.$$

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Warning:  Returning more (or less) than 1 row per `summarise()` group was deprecated in dplyr 1.1.0.
i Please use `reframe()` instead.
i When switching from `summarise()` to `reframe()`, remember that `reframe()` always returns an ungrouped data frame
  and adjust accordingly.
Call `lifecycle::last_lifecycle_warnings()` to see where this warning was generated.
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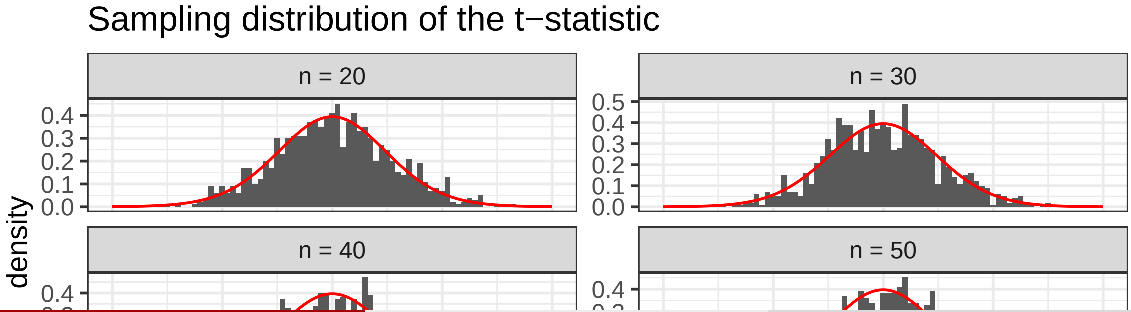
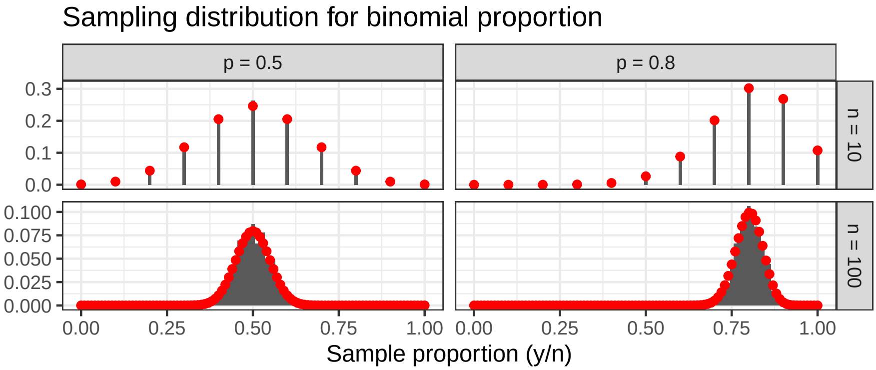


Binomial model

Let $Y \sim Bin(n, p)$, then

$$P\left(\frac{Y}{n} = p\right) = P(Y = np), \qquad p = 0, \frac{1}{n}, \frac{2}{n}, \dots, \frac{n-1}{n}, 1.$$



Approximate sampling distributions

Recall that from the Central Limit Theorem (CLT):

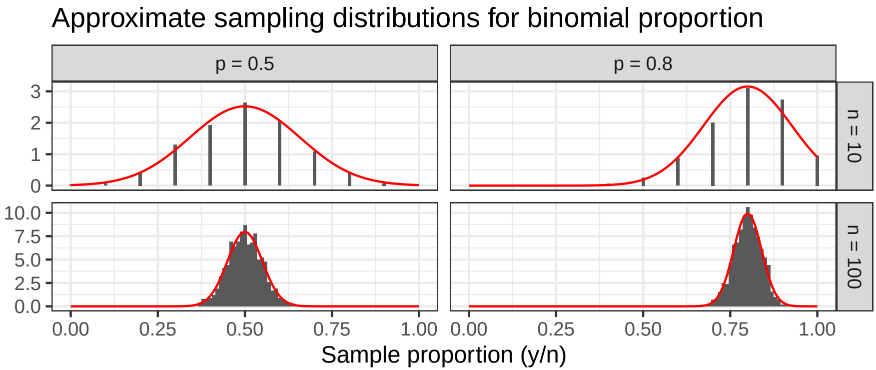
$$S = \sum_{i=1}^{n} X_i \dot{\sim} N(n\mu, n\sigma^2) \qquad \text{and} \qquad \overline{X} = S/n \dot{\sim} N(\mu, \sigma^2/n)$$

for independent X_i with $E[X_i] = \mu$ and $Var[X_i] = \sigma^2$.

Approximate sampling distribution for binomial proportion

If $Y = \sum_{i=1}^{n} X_i$ with $X_i \stackrel{ind}{\sim} Ber(p)$, then

$$\frac{Y}{n} \dot{\sim} N\left(p, \frac{p[1-p]}{n}\right).$$



Summary

Sampling distributions:

- If $Y_i \stackrel{ind}{\sim} N(\mu, \sigma^2)$,
    - $\overline{Y} \sim N(\mu, \sigma^2/n)$ and
    - $\frac{\overline{Y}-\mu}{S/\sqrt{n}} \sim t_{n-1}$.
- If $Y \sim Bin(n, p)$,
    - $P\left(\frac{Y}{n} = p\right) = P(Y = np)$ and
    - $\frac{Y}{n} \dot\sim N\left(p, \frac{p[1-p]}{n}\right)$.
- If X_i independent with $E[X_i] = \mu$ and $Var[X_i] = \sigma^2$, then

$$S = \sum_{i=1}^{n} X_i \dot\sim N(n\mu, n\sigma^2)$$

and

$$\overline{X} = S/n \dot\sim N(\mu, \sigma^2/n)$$

for n sufficiently large.