Why p-values don't mean what you think they mean.

STAT 587 (Engineering)
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p-values

From the ASA statement on p-values:

> *a p-value is the probability under a specified statistical model that a statistical summary of the data would be equal to or more extreme than its observed value.*

Do you have a random sample from your population? If not, then you cannot make a statistically valid statement about the population.

t-test

Let $Y_i \overset{ind}{\sim} N(\mu, \sigma^2)$ with hypotheses

$$H_0 : \mu = \mu_0 \qquad \text{versus} \qquad H_A : \mu \neq \mu_0.$$

Calculate

$$t = \frac{\overline{y} - \mu_0}{s/\sqrt{n}}$$

which has a t_{n-1} distribution if H_0 is true and the associated p-value

$$p\text{-value} = 2P(T \leq -|t|)$$

where $T \sim t_{n-1}$.

You set a significance level a and reject H_0 if

$$p\text{-value} < a.$$

ASA interpretation

From the ASA:

> *a p-value can indicate how incompatible the data are with a specified statistical model*

If you reject H_0, the data are incompatible with the model associated with the null hypothesis. In our example,

$$H_0 : Y_i \overset{ind}{\sim} N(\mu_0, \sigma^2).$$

So perhaps

- the data are not independent,
- the data are not normal,
- the variance is not constant,
- the mean is not μ_0, or
- you got unlucky.

Context matters

From the ASA statement:

> *Scientific conclusions and business or policy decisions should not be based only on whether a p-value passes a specific threshold.*

Imagine these scenarios all with p-value$= 0.05$:

- a small-scale agricultural field trail,
- an extrasensory perception experiment,
- a large-scale clinical trial, or
- a 30,000 gene screening for disease progression.

A simple model

Let $Y \sim N(\mu, 1)$ and $H_0 : \mu = 0$ vs $H_A : \mu \neq 0$.

You observe p-value, $p = 0.05$. What does it mean?

Bayes rule:

$$P(H_0|p = 0.05) = \frac{P(p = 0.05|H_0)P(H_0)}{P(p = 0.05|H_0)P(H_0) + P(p = 0.05|H_A)P(H_A)}$$

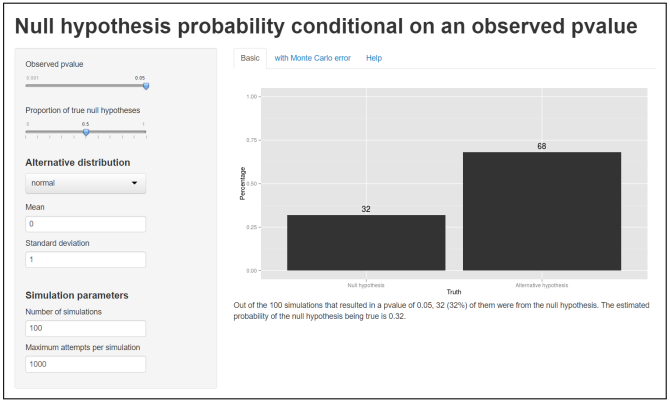
To calculate this we need

- $P(H_0) = 1 - P(H_A)$ and
- $P(p = 0.05|H_A)$:
  - distribution for μ when H_A is true.

p-value shiny app

```
install.packages("shiny")
shiny::runGitHub("jarad/pvalue")
```

https://jaradniemi.shinyapps.io/pvalue/



Summary

- Random sample $\rightarrow$ population
- Model assumptions
- Context matters
- Error rate is likely much larger than significance level