R01a - Simple linear regression:

Choosing explanatory variables

STAT 5870 (Engineering)
Iowa State University

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Simple linear regression

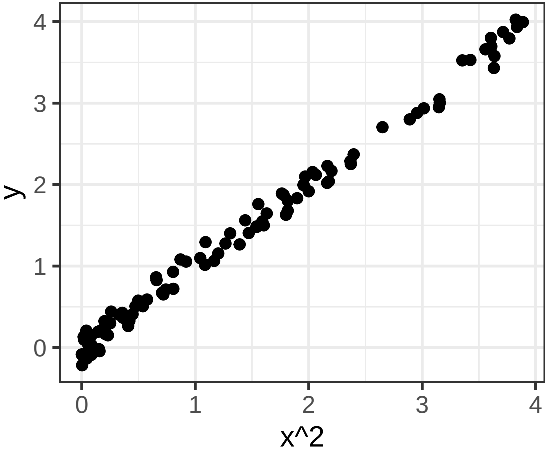
Let

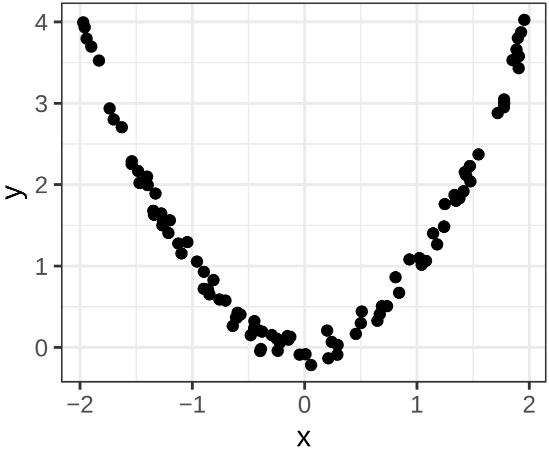
$$Y_i \stackrel{ind}{\sim} N(\beta_0 + \beta_1 f(X_i), \sigma^2).$$

Possible choices for f:

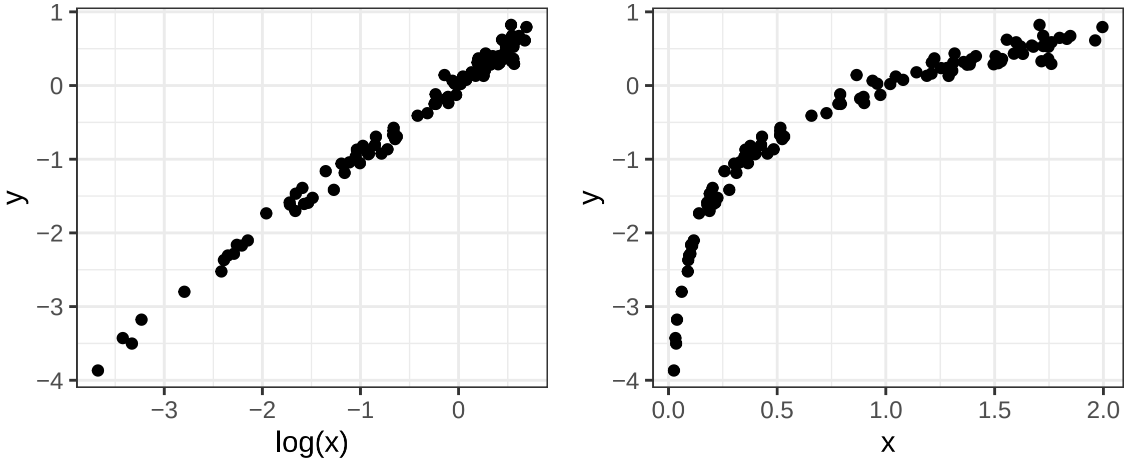
- binary: $f(x) = I(x < c)$
- quadratic: $f(x) = x^2$
- logarithmic: $f(x) = \log(x)$
- centered: $f(x) = x - m$
- scaled: $f(x) = x/s$

Quadratic relationship





Logarithmic relationship



Shifting the intercept

The intercept is the expected response when the explanatory variable is zero. If we use

$$f(x) = x - m,$$

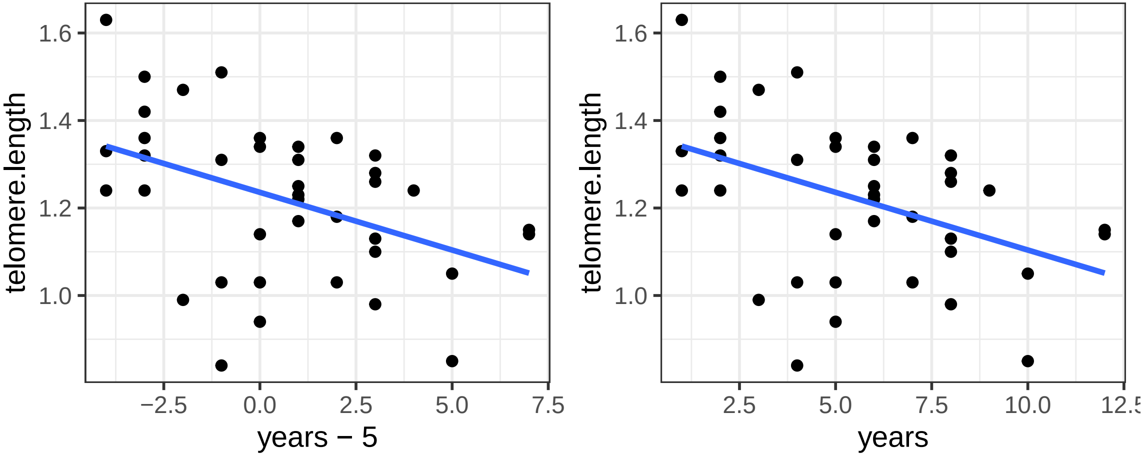
then the new intercept is the expected response when the explanatory variable is m.

$$E[Y|X = x] = \beta_0 + \beta_1(x - m) = \tilde{\beta}_0 + \tilde{\beta}_1 x$$

so our new parameters for the mean are

- slope $\tilde{\beta}_1 = \beta_1$ (unchanged) but
- intercept $\tilde{\beta}_0 = (\beta_0 - m\beta_1)$.

Telomere data



Telomere data: shifting the intercept

```
m0 = lm(telomere.length ~   years    , abd::Telomeres)
m4 = lm(telomere.length ~ I(years-5), abd::Telomeres)

coef(m0)

(Intercept)       years
 1.36768207 -0.02637431

coef(m4)

 (Intercept) I(years - 5)
  1.23581049  -0.02637431

confint(m0)

                  2.5 %       97.5 %
(Intercept)  1.25176134  1.483602799
years       -0.04478579 -0.007962836

confint(m4)

                   2.5 %       97.5 %
(Intercept)   1.18136856  1.290252429
I(years - 5) -0.04478579 -0.007962836
```

Rescaling the slope

The slope is the expected increase in the response when the explanatory variable increases by 1. If we use

$$f(x) = x/s,$$

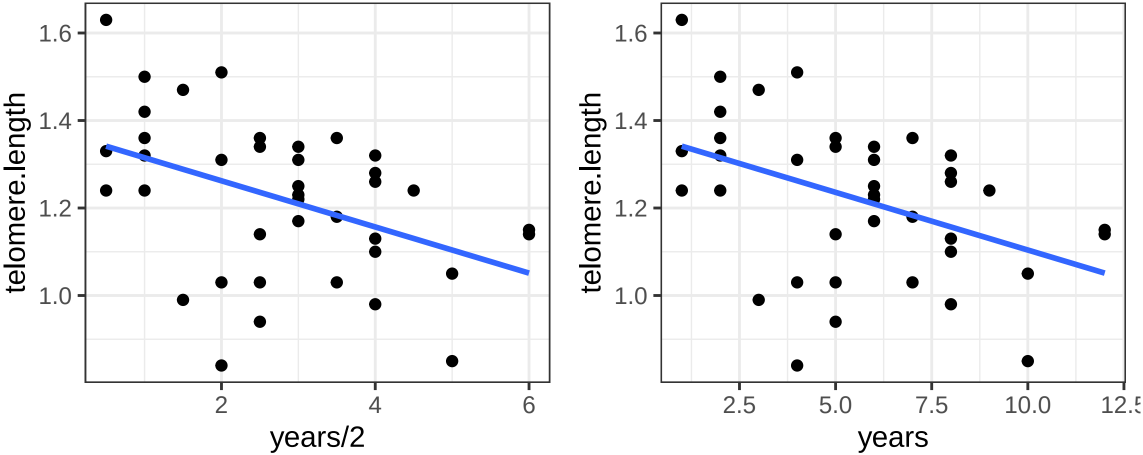
then the new slope is the expected increase in the response when the explanatory variable increases by s.

$$E[Y|X = x] = \beta_0 + \beta_1(x/s) = \tilde{\beta}_0 + \tilde{\beta}_1 x$$

so our new parameters are

- intercept $\tilde{\beta}_0 = \beta_0$ (unchanged) but
- slope $\tilde{\beta}_1 = \beta_1/s$.

Telomere data: rescaling the slope



Telomere data: rescaling the slope

```
m0 = lm(telomere.length ~    years    , abd::Telomeres)
m4 = lm(telomere.length ~ I(years/2), abd::Telomeres)

coef(m0)

(Intercept)       years
 1.36768207 -0.02637431

coef(m4)

(Intercept)  I(years/2)
 1.36768207 -0.05274863

confint(m0)

                  2.5 %       97.5 %
(Intercept)  1.25176134  1.483602799
years       -0.04478579 -0.007962836

confint(m4)

                  2.5 %      97.5 %
(Intercept)  1.25176134  1.48360280
I(years/2)  -0.08957159 -0.01592567
```

Summary

Let

$$Y_i \stackrel{ind}{\sim} N(\beta_0 + \beta_1 f(X_i), \sigma^2).$$

Choose f based on

- Scientific understanding
- Interpretability
- Diagnostics