Dynamic Regression

Dr. Jarad Niemi

STAT 6150 - Iowa State University

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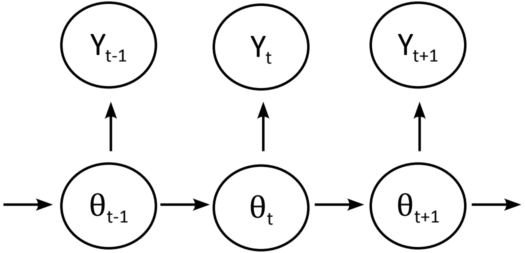
Dynamic Linear Models

$$
\begin{aligned}
Y_t &= F_t \theta_t + v_t & v_t &\stackrel{ind}{\sim} N_m(0, V_t) \\
\theta_t &= G_t \theta_{t-1} + w_t & w_t &\stackrel{ind}{\sim} N_p(0, W_t) \\
& & \theta_0 &\sim N_p(m_0, C_0)
\end{aligned}
$$

where v_t and w_t are independent across time and all are independent of θ_0.



Standard regression model

Consider a temporal regression problem with $y_t \in \mathbb{R}$ and $x_t \in \mathbb{R}^p$. A standard regression model assumes

$$y_t = x_t^\top \theta + \epsilon_t, \qquad \epsilon_t \stackrel{ind}{\sim} N(0, \sigma^2).$$

But, if the effect of x_t on y_t changes with t, we may want to consider the model

$$y_t = x_t^\top \theta_t + \epsilon_t, \qquad \epsilon_t \stackrel{ind}{\sim} N(0, \sigma^2).$$

And put some kind of evolution on θ_t, e.g.

$$\theta_t = G_t \theta_{t-1} + \omega_t, \qquad \omega_t \stackrel{ind}{\sim} N_p(0, W_t).$$

Dynamic regression model

This dynamic regression model written as a DLM is

$$\begin{aligned} Y_t &= F_t\theta_t + v_t & v_t &\stackrel{ind}{\sim} N(0, \sigma_t^2) \\ \theta_t &= G_t\theta_{t-1} + w_t & w_t &\stackrel{ind}{\sim} N_p(0, W_t) \end{aligned}$$

where

- $F_t = x_t^\top$,
- (typically) $G_t = I_p$, and
- (typically) W_t is diagonal.

Background

Use the following notation:

r_t : return at time t of the asset under study

$r_t^{(M)}$: return at time t of the market

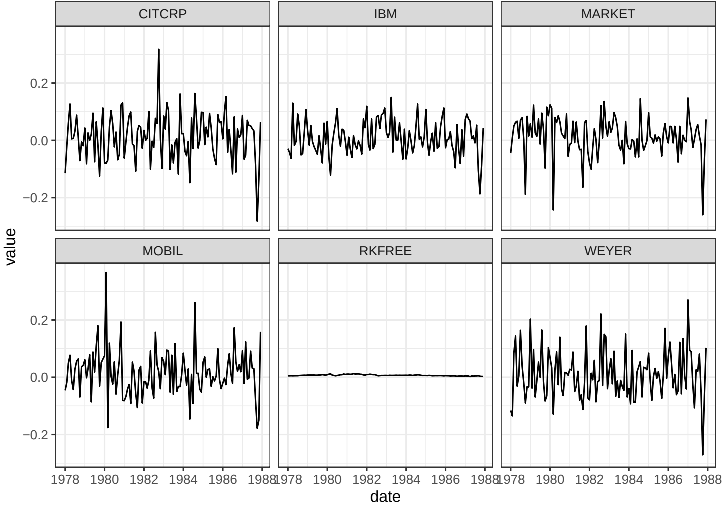
$r_t^{(f)}$: return at time t of a risk free asset

Let

$$\begin{aligned} y_t &= r_t - r_t^{(f)} \\ x_t &= r_t^{(M)} - r_t^{(f)} \end{aligned}$$

A univariate capital asset pricing model (CAPM) assumes that

$$y_t = \alpha + \beta x_t + v_t, \qquad v_t \stackrel{ind}{\sim} N(0, \sigma^2).$$


CITCRP
IBM
MARKET
MOBIL
RKFREE
WEYER
value
0.2
0.0
−0.2
1978
1980
1982
1984
1986
1988
date

Static regression of IBM returns

```
> outLM <- lm(IBM ~ x)
> outLM$coef
  (Intercept)             x
-0.0004895937  0.4568207721
>
> mod <- dlmModReg(x, dV = 0.00254, m0 = c(0, 0),
+                  C0 = diag(c(1e+07, 1e+07)))
> outF <- dlmFilter(IBM, mod)
> outF$m[1 + length(IBM), ]
[1] -0.0004895937  0.4568207719
```

Since the estimate for $\beta < 1$ the stock would be considered conservative.

Dynamic regression of IBM returns

```
> buildCapm <- function(u) {
+     dlmModReg(x, dV = exp(u[1]), dW = exp(u[2 : 3]))
+ }
> outMLE <- dlmMLE(IBM, parm = rep(0, 3), buildCapm)
> exp(outMLE$par)
[1] 2.328402e-03 1.100214e-05 6.495784e-04
> outMLE$value
[1] -276.7014
> mod <- buildCapm(outMLE$par)
> outS <- dlmSmooth(IBM, mod)
> plot(as.zoo(dropFirst(outS$s)), main = "",
       mar = c(0, 2.1, 0, 1.1),
       oma = c(2.1,0,.1,.1), cex.axis = 0.5)
```

CAPM example